Public Announcements for Epistemic Models and Hypertheories

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Outline

Logic K_n: Syntax and Semantics

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Extension of the classical propositional logic with modalities $\Box_i$, for $i = 1, \dots, n$. Let $\mathsf{Prop} = \{p, q, \dots\}$,

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The semantics of K_n is standard Kripke semantics. Model is a tuple $\mathcal{K} = (W, R_1, \ldots, R_n, \Vdash)$, where

K1) $W \neq \emptyset$;

K2) R_i is a binary relation on W, for $i = 1, \ldots, n$;

K3) $\Vdash$: Prop $\to 2^W$.

One Observation

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In Epistemic Models, the condition (1) is replaced by a weaker condition:

$$u \models \Box_i A \;\Rightarrow\; \forall v(R_i(u,v) \Rightarrow v \models A).$$

Closure of a Set of Formulas and Belief Sets

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Definition

Let L be a logic and T be a set of formulas.

1. $cl^0_{\mathsf{L}}(T) = T \cup \{A \mid \mathsf{L} \vdash A\}$;
2. $cl^{j+1}_{\mathsf{L}}(T) = cl^j_{\mathsf{L}}(T) \cup \{A \mid B \in cl^j_{\mathsf{L}}(T)$ and $B \to A \in cl^j_{\mathsf{L}}(T)$, for some $B\}$;
3. $F \in cl_{\mathsf{L}}(T)$ iff $F \in cl^j_{\mathsf{L}}(T)$, for some j.

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3. $F \in cl_{\mathsf{L}}(T)$ iff $F \in cl^j_{\mathsf{L}}(T)$, for some j.

Definition (Belief Set)

For a given logic L, an L-*belief set* is a set of formulas T with $T = cl_{\mathsf{L}}(T)$.

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For a set $Z \subseteq X \times Y$ and an element $x \in X$, we set $(Z)_x := \{y \mid (x, y) \in Z\}$.

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Definition (Pre-epistemic Model)

Let L be a logic. A *pre-epistemic* L-*model* is a tuple $\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu^1_{\mathcal{B}}, \ldots, \nu^n_{\mathcal{B}})$, where:

- $W \neq \emptyset$ is a non-empty set of states;
- $R_1, \ldots, R_n$ are binary relations on W;
- $\nu \subseteq W \times \mathrm{Prop}$;
- $\nu^i_{\mathcal{B}} \subseteq W \times \mathrm{For}$, such that for every $u \in W$, $(\nu^i_{\mathcal{B}})_u$ is an L-belief set.

Satisfaction Relation and Epistemic Model

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Definition (Satisfaction Relation in Pre-epistemic Model)

Let $\mathcal{E}$ be a pre-epistemic L-model and $u \in W$. The satisfaction relation, $\models$, is defined as follows:

- $\mathcal{E}, u \models p$ iff $(u, p) \in \nu$;
- $\mathcal{E}, u \models A \wedge B$ iff $\mathcal{E}, u \models A$ and $\mathcal{E}, u \models B$;
- $\mathcal{E}, u \models \neg A$ iff $\mathcal{E}, u \not\models A$;
- $\mathcal{E}, u \models \Box_i A$ iff $(u, A) \in \nu^i_{\mathcal{B}}$.

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Definition (Epistemic Model)

Let L be a logic. An *epistemic* L-*model* is a pre-epistemic L-model that satisfies

$$\mathcal{E}, u \models \Box_i A \;\Rightarrow\; \mathcal{E}, R_i(u) \models A, \tag{2}$$

where $\mathcal{E}, R_i(u) \models A$ stands for $\forall v(R_i(u, v) \Rightarrow \mathcal{E}, v \models A)$.

Embedding Theorem

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Theorem

For any epistemic K_n*-model*

$$\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu^1_{\mathcal{B}}, \ldots, \nu^n_{\mathcal{B}}),$$

there exists a Kripke model

$$\mathcal{K} = (\widetilde{W}, \widetilde{R_1}, \ldots, \widetilde{R_n}, \Vdash),$$

such that:

(a) $W \subseteq \widetilde{W}$;

(b) $R_i \subseteq \widetilde{R}_i$;

(c) *for each* $u \in W$ *and each formula* A,

$$\mathcal{E}, u \models A \qquad \textit{iff} \qquad \mathcal{K}, u \Vdash A.$$

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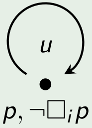

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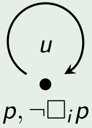

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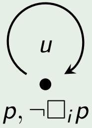

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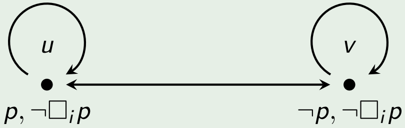

Hypertheories

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Definition (Hypertheory)

A *hypertheory* is a tuple

$$\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T}),$$

where:

- $(W, R_1, \ldots, R_n)$ is a frame;
- $\mathcal{T}$ assigns a set of formulas T_u to each $u \in W$.

Hypertheories ctd.

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Definition

An epistemic L-model $\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu_{\mathcal{B}}^1, \ldots, \nu_{\mathcal{B}}^n)$ is a *model of* $\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T})$ if for each $u \in W$,

$$u \models T_u \quad (\text{i.e. } u \models A, \text{ for each } A \in T_u).$$

A formula A *logically* L-*follows* from a hypertheory $\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T})$ at state $u \in W$, denoted by

$$\mathcal{H}, u \models_{\mathsf{L}} A,$$

if $\mathcal{E}, u \models A$ for each epistemic L-model $\mathcal{E}$ of $\mathcal{H}$.

Hypertheories ctd.

Definition (Hyperderivation)

Let $\mathcal{H}$ be a hypertheory. A formula A is L-*hyperderivable* at $u \in W$ (write $\mathcal{H}, u \Vdash_{\mathsf{L}} A$) if A can be obtained by the rules:

1) classical inference[a]:
   a) $u \Vdash_{\mathsf{L}} A$, if $A \in T_u$;
   b) $u \Vdash_{\mathsf{L}} A$, if $\mathsf{L} \vdash A$;
   c) $u \Vdash_{\mathsf{L}} A$, if $u \Vdash_{\mathsf{L}} B \rightarrow A$ and $u \Vdash_{\mathsf{L}} B$ for some formula B;
2) transition: $u \Vdash_{\mathsf{L}} \Box_i A \;\Rightarrow\; R_i(u) \Vdash_{\mathsf{L}} A$;
3) deduction: $u \cup A \Vdash_{\mathsf{L}} B \;\Rightarrow\; u \Vdash_{\mathsf{L}} A \rightarrow B$, where for a hypertheory $\mathcal{H}$, $\mathcal{H}^{v \cup A}$ is defined as a hypertheory $\mathcal{H}$ where T_v is replaced by $T_v \cup \{A\}$ and $u \cup A \Vdash_{\mathsf{L}} B$ stands for $\mathcal{H}^{u \cup A}, u \Vdash_{\mathsf{L}} B$;
4) consistency: if uR_iv, then $v \Vdash_{\mathsf{L}} \bot \;\Rightarrow\; u \Vdash_{\mathsf{L}} \bot$.

[a] As usual, we write $u \Vdash_{\mathsf{L}} A$ instead of $\mathcal{H}, u \Vdash_{\mathsf{L}} A$ when $\mathcal{H}$ is clear from the context.

Soundness and Completeness

Theorem

For a hypertheory $\mathcal{H}$ and any Fml*-formula A,*

$$\mathcal{H}, u \Vdash_{\mathsf{K}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{K}_n} A.$$

Outline

1. Artemov's Work on Epistemic Models and Hypertheories

2. Public announcements
   - Case of Belief Change in Epistemic Models: Logic KPA_n
   - Case of Knowledge Change in Epistemic Models

Logic KPA_n

The logic KPA_n is an extenstion of the logic K_n with an announcement operator $[\cdot]$. The set of formulas $\mathsf{Fml}_{[\cdot]}$ is generated by the following grammar:

$$A := p \mid \neg A \mid (A \wedge A) \mid \Box_i A \mid [A]A.$$

Axiomatization of KPA_n

Axiom schemes:

B1) all instantiations of classical propositional tautologies

B2) $\Box_i(A \to B) \to (\Box_i A \to \Box_i B)$

B3) $[A]p \leftrightarrow p$

B4) $[A]\neg B \leftrightarrow \neg[A]B$

B5) $[A](B \wedge C) \leftrightarrow ([A]B \wedge [A]C)$

B6) $[A]\Box_i B \leftrightarrow \Box_i(A \to B)$

B7) $[A][B]C \leftrightarrow [A \wedge B]C$

Inference rules:

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It is clear that we have the usual "rewriting" property for public announcements, which makes it possible to remove all announcements from an arbitrary formula.

Satisfaction Relation

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We add the following clause:

- $\mathcal{E}, u \models [A]B$ iff $\mathcal{E}|_A, u \models B$,

where the restricted model $\mathcal{E}|_A = (W', R'_1, \ldots, R'_n, \nu', \nu'^1_{\mathcal{B}}, \ldots, \nu'^n_{\mathcal{B}})$ is given by:

$$W' = W,$$
$$R'_i = R_i \cap ([\![A]\!]_{\mathcal{E}} \times [\![A]\!]_{\mathcal{E}}),$$
$$\nu' = \nu,$$
$$\nu'^i_{\mathcal{B}} = \{(w, B) \mid w \in W \text{ and } B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_w \cup \{A\})\},$$

with $[\![A]\!]_{\mathcal{E}} = \{w \in W \mid \mathcal{E}, w \models A\}$.

Some Lemmas

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Lemma

For any formula A, if $\mathcal{E}$ is an epistemic KPA_n*-model, then the restricted model* $\mathcal{E}|_A$ *is an epistemic* KPA_n*-model, too.*

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For any formula A, if $\mathcal{E}$ is an epistemic KPA_n-model, then the restricted model $\mathcal{E}|_A$ is an epistemic KPA_n-model, too.

Note that KPA_n is a conservative extension of the modal logic K_n with respect to announcement-free formulas. We have for each Fml-formula A,

$$\mathsf{KPA}_n \vdash A \quad \text{iff} \quad \mathsf{K}_n \vdash A.$$

This conservativity result can be transfered to logical consequence.

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This conservativity result can be transfered to logical consequence.

Lemma

Let $\mathcal{H}$ be a hypertheory consisting of Fml*-formulas and let u be a state of $\mathcal{H}$. We have for each* Fml*-formula A,*

$$\mathcal{H}, u \models_{\mathsf{KPA}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{K}_n} A.$$

Soundness and Completeness

Using previous Lemma and "rewriting property" we can do the usual completeness by reduction proof for KPA_n.

Theorem (Soundness and Completeness Theorem)

Let $\mathcal{H}$ be a hypertheory containing Fml*-formulas. For each* $\mathsf{Fml}_{[\cdot]}$*-formula A, we have*

$$\mathcal{H}, u \Vdash_{\mathsf{KPA}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{KPA}_n} A.$$

AGM Postulates

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First of all, we have that all announcements are *successful*, i.e., after any announcement of A, each agent beliefs A. Formally, we have that

$$[A]\Box_i A$$

is KPA_n-valid.

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$$[A]\Box_i A$$

is KPA_n-valid.
Further, we have *persistence of beliefs*, i.e, no announcement will change existing beliefs. The formula

$$\Box_i A \to [B]\Box_i A$$

is KPA_n-valid.

Minimal Change

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Lemma (Minimal Change)

Let $\mathcal{E}$ be an epistemic KPA_n*-model with a state w. Let $\mathcal{F}$ be any epistemic* KPA_n*-model with a state v such that*

1. $\mathcal{F}, v \models \Box_i A$
2. $\mathcal{E}, w \models \Box_i B$ *implies* $\mathcal{F}, v \models \Box_i B$ *for all formulas B.*

Then we find that for all formulas B,

$$\mathcal{E}|_A, w \models \Box_i B \text{ implies } \mathcal{F}, v \models \Box_i B.$$

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Truthful Public Announcements over $S4_n$ (Kripkean case)

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For a given Kripke model $\mathcal{M} = (W, R_1, \ldots, R_n, \Vdash)$, satisfiability of announcement formulas is defined by

$$\mathcal{M}, s \Vdash [A]B \quad \text{iff} \quad \mathcal{M}, s \Vdash A \ \text{ implies } \ \mathcal{M}|_A, s \Vdash B, \tag{3}$$

where $\mathcal{M}|_A = (W', R'_1, \ldots, R'_n, \Vdash')$ is the restriction of the model defined as

$$W' = [\![A]\!]_{\mathcal{M}},$$
$$R'_i = R_i \cap ([\![A]\!]_{\mathcal{M}} \times [\![A]\!]_{\mathcal{M}}),$$
$$\Vdash' = \Vdash \cap [\![A]\!]_{\mathcal{M}},$$

for $[\![A]\!]_{\mathcal{M}} = \{w \in W \mid w \Vdash A\}$.

Truthful Public Announcements over $S4_n$ (Epistemic case)

Truthful Public Announcements over $\mathsf{S4}_n$ (Epistemic case)

Adapting this strategy for an epistemic $\mathsf{S4}_n$-model

$$\mathcal{E} = (W, R_1, \dots, R_n, \nu, \nu^1_{\mathcal{B}}, \dots, \nu^n_{\mathcal{B}})$$

yields the following definition of satisfiability:

$$\mathcal{E}, s \models [A]B \quad \text{iff} \quad \mathcal{E}, s \models A \ \text{ implies } \ \mathcal{E}|_A, s \models B,$$

where $\mathcal{E}|_A$ is given by

$$\begin{aligned}
&W' = [\![A]\!]_{\mathcal{E}},\\
&R'_i = R_i \cap ([\![A]\!]_{\mathcal{E}} \times [\![A]\!]_{\mathcal{E}}),\\
&\nu' = \nu,\\
&\nu'^i_{\mathcal{B}} = \{(w, B) \mid w \in [\![A]\!]_{\mathcal{E}} \text{ and } B \in cl_{\mathsf{S4}_n}((\nu^i_{\mathcal{B}})_w \cup \{A\})\},
\end{aligned}$$

where cl_{S4_n} is given as in Definition 1 with the addition of

- if $A \in cl^j_{\mathsf{S4}_n}(T)$, then $\Box_i A \in cl^{j+1}_{\mathsf{S4}_n}(T)$,
- if $\Box_i A \in cl^j_{\mathsf{S4}_n}(T)$, then $A \in cl^{j+1}_{\mathsf{S4}_n}(T)$.

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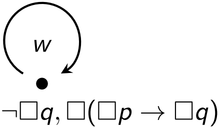
Consider $\mathcal{E} = (W, R, \nu, \nu_{\mathcal{B}})$ with

$$W = \{w\},\ R = \{(w, w)\},\ \nu = \{(w, p)\},$$
$$\nu_{\mathcal{B}} = \{(w, A) \mid A \in cl_{S4_n}(\{\Box p \to \Box q\})\}.$$



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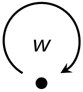

$$p, \neg q, \neg\Box p, \neg\Box q, \Box(\Box p \to \Box q)$$

Since p holds at the state w, the restriction of $\mathcal{E}$ to the formula p yields

$$\mathcal{E}|_p = (W, R, \nu, \nu'_{\mathcal{B}}) \text{ with } \nu'_{\mathcal{B}} = \{(w,A) \mid A \in cl_{\mathsf{S4}_n}(\{\Box p \to \Box q, p\})\}.$$

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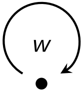

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Thus, by the closure conditions of cl_{S4_n}, we get $\Box p \in (\nu'_{\mathcal{B}})_w$, thus $\Box q \in (\nu'_{\mathcal{B}})_w$ and finally $q \in (\nu'_{\mathcal{B}})_w$. However, now we have the situation that $\mathcal{E}|_p, w \models \Box q$ but also $\mathcal{E}|_p, w \not\models q$.

Furtherwork

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Does the Soundness and Completeness theorem hold
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Provide a satisfying framework in the case of knowledge

Thank You!