Propositional modal logic with implicit modal quantification

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3-March-2019
ICLA
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  $\Box_G \alpha$ where $G \subseteq \{1 \cdots n\}$
- Agent set has to be finite and fixed a priori.
  Is there a way to relax this condition?

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- Term modal logic (TML) (Fitting, Thalmann and Vorankov[2001]) offers another alternative.
- In TML modalities are indexed by terms and these terms can be quantified over.
  Ex: $\exists x \Box_x \alpha$.
- But this brings in variables into the picture and makes the syntax and semantics complex.
  Is there some way to quantify over the agent set without deviating much from the classical propositional modal logics?

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- New modal operators:
  $[\forall]\alpha$ to mean for all agents *a*, $\Box_a\alpha$
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- These modal operators implicitly quantify over the agent set and hence the name Implicitly quantified modal logic.

IQML syntax

Let $\mathcal{P}$ be a countable set of propositions. The syntax of IQML is given by:

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Duals: $\langle\forall\rangle\phi$ and $\langle\exists\rangle\phi$.

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$M, w \models [\exists]\phi \quad \Leftrightarrow \quad$ there is some $i \in \mathbb{I}$ such that for all $u \in \mathcal{W}$ if $(w, u) \in R_i$ then $\mathcal{M}, u \models \phi$

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\mathcal{M}, w \models p & \Leftrightarrow & p \in \rho(w) \\
M, w \models [\exists]\phi & \Leftrightarrow & \text{there is some } i \in \mathbb{I} \text{ such that for all } u \in \mathcal{W} \\
& & \text{if } (w, u) \in R_i \text{ then } \mathcal{M}, u \models \phi \\
\mathcal{M}, w \models [\forall]\phi & \Leftrightarrow & \text{for all } i \in \mathbb{I} \text{ and for all } u \in \mathcal{W} \\
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Properties of modalities

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Theorem

IQML *admits a sound and complete axiomatization.*

Bisimulation for ML

- Given 2 Kripke models $M = (W, R_1 \cdots R_n, \rho)$ and $M' = (W', R'_1 \cdots R'_n, \rho')$ the relation $B \subseteq (W \times W')$ is a bisimulation if for all $(w, w') \in B$ and for all $i \leq n$:

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$$\rho(w) = \rho'(w')$$

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Bisimilar worlds agree on all modal formulas.

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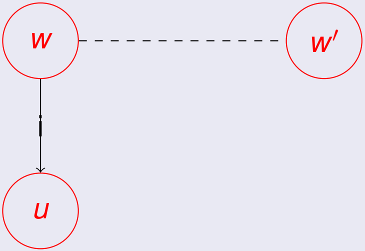

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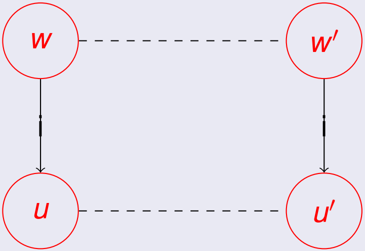

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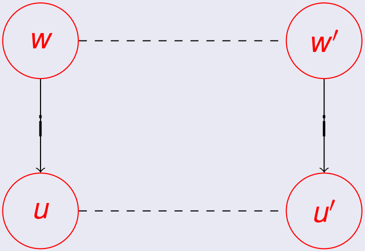

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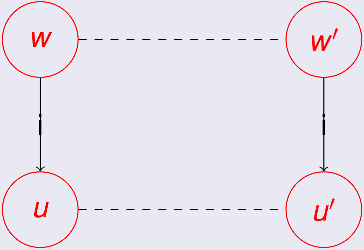

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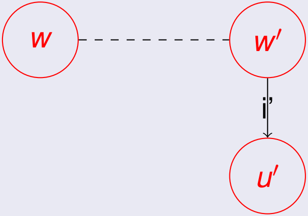

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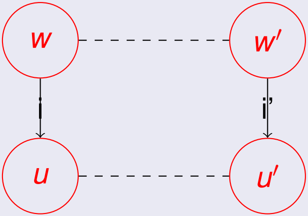

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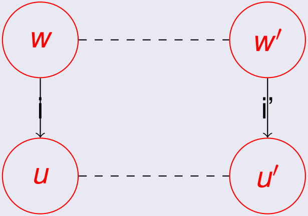

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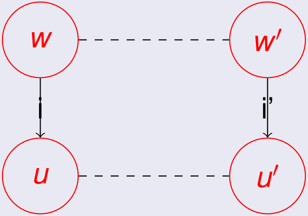

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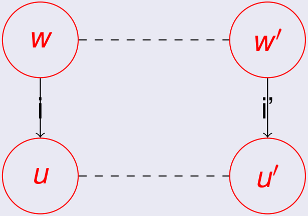

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Let $\phi(x) \in$ *2-sorted* FO *with one free variable, then:*
$\phi(x)$ *is* IQML*-bisimulation invariant iff* $\phi(x)$ *is equivalent to some* IQML *formula.*

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  - Also, we need a d_β successor which satisfies β, ϕ and ψ.
  - Finally for each $e_\gamma \in I$ we need a new successor which satisfiesϕ and ψ.

Extension to bundled fragment of TML

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- Decidability result holds even with predicates of arbitrary arity.

  - Unrestricted version is undecidable with atoms restricted to propositions.

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